A representation of the Heisenberg group by operators acting on phase space; applications

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The Heisenberg–Weyl operators $\widehat{T}(z_0) = e^{-\frac{i}{\hbar}\sigma(\hat{z},z_0)}$, which act on $L^2(\mathbb{R}^n)$, lead to an irreducible representation (the "Schrödinger representation") on $L^2(\mathbb{R}^n)$ of the Heisenberg group $\mathbb{H}_n$. The Schrödinger representation leads to the Weyl quantization procedure associating to a symbol a an operator $\widehat{A} = a(x, -i\partial_x)$. The Stone–von Neumann theorem is often invoked to claim that this is the only possible irreducible representation of $\mathbb{H}_n$. In this talk we show that there is a class of operators $\widetilde{T}(z_0)$ acting on $L^2(\mathbb{R}^n \oplus \mathbb{R}^n)$ and corresponding to infinitely many intertwined representations of $\mathbb{H}_n$ on closed subspaces of $L^2(\mathbb{R}^n \oplus \mathbb{R}^n)$. The operators $\widetilde{T}(z_0)$ lead to phase-space pseudo-differential operators formally given by $\widetilde{A} = a\left(x + \frac{1}{2}i\partial_y, y - \frac{1}{2}i\partial_x\right)$ ("Bopp operators"). These operators are intertwined with $\widehat{A} = a(x, -i\partial_x)$ by an infinite family of partial isometries $L^2(\mathbb{R}^n) \longrightarrow L^2(\mathbb{R}^n \oplus \mathbb{R}^n)$. We study the spectral properties of $\widetilde{A}$ when the symbol a belongs to a certain Shubin class, and apply our results to two examples: magnetic operators and Moyal's star-product.

This talk is based on the papers:

(1) M. de Gosson: Spectral Properties of a Class of Generalized Landau Operators. *Communications in Partial Differential Equations*, **33**(11) 2008.
(2) M. de Gosson and F. Luef: Spectral and Regularity properties of a Pseudo-Differential calculus Related to Landau Quantization. *Journal of Pseudo-Differential Operators and Applications* **1**(1), 2010.