The Cauchy–Kovalevskaya extension in Hermitean Clifford analysis

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The Cauchy–Kovalevskaya extension theorem is well–known: in particular, it follows from this theorem that a holomorphic function in an appropriate region of the complex plane is completely determined by its restriction to the real axis. This holomorphic CK–extension principle has been elegantly generalized to higher dimension in the framework of Clifford analysis, a higher dimensional function theory centered around the notion of a monogenic function, i.e. a Clifford algebra valued null solution of the Dirac operator $\underline{\partial} = \sum_{j=1}^{m} e_j \, \partial_{x_j}$, where $(e_1, \ldots, e_m)$ is an orthonormal basis of $\mathbf{R}^m$ underlying the construction of the real Clifford algebra $\mathbf{R}_{0,m}$. Completely similar to the holomorphic case, a monogenic function in an appropriate region of $\mathbf{R}^m$ will be completely determined by its restriction to the hyperplane $x_1 = 0$.

More recently Hermitean Clifford analysis has emerged as a new branch of Clifford analysis, which focusses on the simultaneous null solutions, called Hermitean monogenic functions, of two Hermitean conjugate complex Dirac operators. The functions considered now take their values in the complex Clifford algebra $\mathbf{C}_m$. In this talk we establish a Cauchy–Kovalevskaya extension theorem for Hermitean monogenic polynomials. The minimal number of initial polynomials needed to obtain a unique extension is determined, along with the compatibility conditions they have to satisfy.

This is joint work with F. Brackx, R. Lavička and V. Souček.