Self-adjoint linearizations of eigenvalue problems with boundary conditions which depend polynomially on the eigenvalue parameter

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The eigenvalue problems we consider are of the form

$$S^* f = \lambda f, \quad P(\lambda)\mathrm{b}(f) = 0,$$

where S^* is the adjoint of a densely defined symmetric operator S in a Hilbert space with both defect numbers equal to d, b is a boundary mapping on the domain of S^* with values in $\mathbb{C}^{2d}$ and corresponding $2d \times 2d$ Gram matrix Q, and $P(z)$ is a $d \times 2d$ matrix polynomial such that

1. $P(z)Q^{-1}P(z^*)^* = 0$ for all $z \in \mathbb{C}$,
2. $P(z)$ has rank d for all $z \in \mathbb{C}$, and
3. for some nonnegative integers $\mu_1, \ldots, \mu_d$ the limit

   $$P_\infty := \lim_{z\to\infty} \operatorname{diag}\left(z^{-\mu_1}, \ldots, z^{-\mu_d}\right) \mathcal{P}(z)$$

   exists and is a $d \times 2d$ matrix of rank d.

In the lecture we define and investigate the self-adjoint linearization of this problem and discuss corresponding eigenfunction expansions.

The lecture is based on joint work in progress with T. Azizov and B. Curgus.