Similarity, dilations, and commutant lifting in the DA-Space

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During the past fifty years, an extensive and refined theory has been developed based on a canonical model for contraction operators on Hilbert space. A key ingredient in this development was the commutant lifting theorem (CLT), which grew out of a seminal paper of Sarason. In the last decade or two, there has been a systematic effort to extend much of this theory to the Drury-Arveson (DA) space in m variables, including the CLT. The latter development is due to a number of authors including Arveson, McCullough-Trent, Muller-Vasilescu, and Popescu.

In this talk, based on joint work with Foias and Sarkar, and expressed in the language of Hilbert modules, I will discuss results on similarity of quotient Hilbert modules of vector-valued DA-modules with the splitting of the corresponding short exact sequences and the complementation of the submodules, including connections to the CLT. Furher, I will show that all isometries on vector-valued DA-modules are trivial, in a sense to be defined, and use this fact to begin the analysis of more general resolutions of such modules and discuss their relation to dilations.