Extension of functions from hypersurfaces with the boundary

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A coretraction, a right inverse to the trace operator to a smooth hypersurface $\mathcal{S}$ with the smooth boundary $\Gamma = \partial\mathcal{S} \neq \emptyset$, is constructed. The coretraction extends m-tuples of functions from the Besov spaces on each face of $\mathcal{S}$ into the Bessel potential space on the domain slit by the hypersurface $\mathbb{R}^n_{\mathcal{S}} := \mathbb{R}^n \setminus \overline{\mathcal{S}}$, provided these tuples satisfy a compatibility conditions on the boundary Γ. The traces are defined by arbitrary Dirichlet system of boundary operators and extension is performed by two different methods, one implicit and one explicit. Explicit extension, based on the solution to the Dirichlet BVP for the poly harmonic equation, permits the extension of distributions from the Besov space $\mathbb{B}^s_{p,p}(\mathcal{S})$ with a negative $s < 0$. Moreover, it permits to establish some additional features of the extended functions, which are useful in applications.

Coretractions have essential applications in boundary value problems for partial differential equations when, for example, it is necessary to reduce a BVP with non-homogeneous boundary conditions to a BVP with the homogeneous boundary conditions.

For a pair of Besov spaces we introduce the following shortcut $\overset{2}{\mathbb{B}}{}^s_{p,p}(\mathcal{S}) := \mathbb{B}^s_{p,p}\mathcal{S}) \otimes \widetilde{\mathbb{B}}^s_{p,p}(\mathcal{S})$ and denote $p_s := p$ if $s \neq 0, \pm1, \dots$, $p_s < p$ if $s = 0, \pm1, \dots$. The notation $[s]^- \in \mathbb{Z}$ is used for the largest positive or negative integer less than s, i.e., $s - 1 \leqslant [s]^- < s$.

Here is one of the principal results obtained in the present investigation,

THEOREM. *Let $\mathcal{S}$ be a smooth hypersurface with the boundary, $\mathbf{A}(x, D)$ be a PDO of order $k \in \mathbb{N}_0$ and of normal type, $1 < p < \infty$ and $k \in \mathbb{N}_0$, $s > 0$, $k \leqslant s + 1$. Further let*

$$\vec{\mathbf{B}}^{(k)}(x, D) := \{\mathbf{B}_0(x, D), \dots, \mathbf{B}_{k-1}(x, D)\}^\top$$

be a Dirichlet system of boundary operators and $\left\{\varphi_j^{\pm}\right\}_{j=0}^{k-1}$ *be vector functions such that*

$$\Phi_j := (\varphi_j^{+} + \varphi_j^{-}, \varphi_j^{+} - \varphi_j^{-}) \in \overset{2}{\mathbb{B}}{}_{p,p}^{s-j}(\mathcal{S}), \qquad \text{for all} \quad j = 0, 1, \ldots, k-1.$$

Then there exists a continuous linear operator

$$\mathcal{P}_{\mathbf{A}} \ : \ \bigotimes_{j=1}^{k-1} \overset{2}{\mathbb{B}}{}_{p,p}^{s-j}(\mathcal{S}) \to \mathbb{H}_{p_s,loc}^{s+1/p_s}(\mathbb{R}_{\mathcal{S}}^n)$$

which has the prescribed traces on the boundary

$$\gamma_{\mathcal{S}^{\pm}} \mathbf{B}_j \mathcal{P}_{\mathbf{A}} \Phi = \varphi_j^{\pm}, \quad j = 0, 1, \ldots, k-1, \qquad \mathbf{A} \mathcal{P}_{\mathbf{A}} \Phi \in \widetilde{\mathbb{H}}_{p_s,loc}^{s-k+1/p_s}(\mathbb{R}_{\mathcal{S}}^n)\,,$$

where $\Phi := \left\{\Phi_j\right\}_{j=0}^{k-1}$.