Quantum mechanical methods in electromagnetism

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In physics, Wigner Quantum Systems play an important role in the study of the physical model of the harmonic oscillator described as a superposition of $n-$independent Hamiltonians of the form$-\frac{1}{2m}\mathbf{a}_j 2 + \frac{m\omega 2}{2}(\mathbf{a}_j^\dagger)2$, where m denotes the mass and ω denotes the frequency.

According to Wigner approach [5], Wigner Quantal Systems are canonically equivalent to the Lie superalgebra of the type $\mathrm{osp}(1|2n)$. This, in particular, allows a meaningful description of discrete function theory based in terms of Wigner Quantal Systems as representations of $\mathrm{osp}(1|2)$ [2].

There is another approach developed by G-C. Rota and collaborators, where the description of umbral calculus was obtained by means of bosonic calculus in interplay with the second quantization approach [1]. Indeed, the algebra of multivariate polinomials is isomorphic to the free algebra generated by position and momentum operators, $\mathbf{a}_j^\dagger$ and $\mathbf{a}_j$, respectively, satisfying the Heisenberg-Weyl relations

$$[\mathbf{a}_j, \mathbf{a}_k] = 0, \quad [\mathbf{a}_j^\dagger, \mathbf{a}_j^\dagger] = 0, \quad [\mathbf{a}_j, \mathbf{a}_k^\dagger] = \delta_{jk}\mathbf{id}$$

In this talk, it will be presented a similar approach to the approach recently proposed by N. Faustino e G.Ren in [3] for the time-harmonic Maxwell equations.

We will start to make an overview for the formulation of the Maxwell equations in the 3D case as well as the topological and material laws encoded.

Next, using the machinery already developed in [2], we will describe the polynomial solutions of the Maxwell time-harmonic equations by means of Wigner Quantal Systems.

The talk is based on a joint work with D. Constales and R.S. Kraußhar.

References

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[4] Constales D., Faustino N., Kraußhar R.S. *Displaced (poly-)Bargmann spaces and the regular solutions of time-harmonic Maxwell equations*, in preparation

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