The role of positivity in moment and polynomial optimization problems

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Let $y \equiv y^{(2d)} = \{y_i\}_{i \in \mathbb{Z}_+^n, |i| \leq 2d}$ denote an n-dimensional real multisequence of degree $2d$ with $y_0 > 0$, and let K denote a closed subset of $\mathbb{R}^n$. We consider connections between the following two classical problems: 1) (Truncated K-Moment Problem) Find conditions for the existence of a positive Borel measure μ, $supp\ \mu \subseteq K$, such that $y_i = int_K x^i d\mu \quad (|i| \leq 2d)$; 2) (Polynomial Optimization Problem) For a given polynomial $p \in \mathbb{R}[x_1, \ldots, x_n]$, compute (or estimate) $p_* := \inf_{x \in K} p(x)$. These problems lead naturally to various notions of positivity: positive semidefiniteness of the moment matrix and K-localizing matrices associated with y; K-positivity of the Riesz functional L_y, defined by

$$L_y \sum_{|i| \leq 2d} a_i x^i = \sum a_i y_i;$$

and positivity of polynomials on K. We discuss recent results which shed new light on the interconnectedness of these concepts and on their roles in solving moment and optimization problems. This talk is based in part on joint work with R. Curto, with C. Easwaran, and with J. Nie.