# Equivalence of the indefinite Sturm-Liouville Riesz basis property with a HELP-type inequality

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For an indefinite weight function $r \in L^1[-1,1]$ with $xr(x) > 0$ we consider two problems which at first glance seem to be different. The first is the Riesz basis property of the indefinite Sturm-Liouville eigenvalue problem

$$-f'' = \lambda r f, \qquad f(-1) = f(1) = 0.$$

The second problem is the validity of the HELP-type inequality on $[0,1]$

$$\left( \int_0^1 |h'|^2 \frac{1}{r}\, dx \right)^2 \leq K \left( \int_0^1 |h|^2 dx \right) \left( \int_0^1 | \left( \frac{h'}{r} \right)' |^2 dx \right)$$

for a certain class of functions $h$. We show that for so-called strongly odd dominated functions $r$ (including at least all odd $r$) both problems are equivalent. This allows us to apply known results from the theory of one problem to the other. The talk is based on a joint work with P. Binding.