Tensor products of GB^*-algebras and applications

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GB^*-algebras are generalizations of C^*-algebras. They were introduced and studied first by G.R. Allan, in 1967. In 1970, P.G. Dixon extended the concept of a GB^*-algebra, in order to include also topological $*$-algebras which are not locally convex. The importance of GB^*-algebras is mainly due to the fact that being algebras of unbounded operators have interesting applications in mathematical physics and this gives a strong impetus for studying them. Typical examples of GB^*-algebras are pro-C^*-algebras (i.e, inverse limits of C^*-algebras), C^*-like locally convex $*$-algebras (introduced by A. Inoue - K.-D. Kürsten), the Arens algebra $L^\omega[0,1] = \bigcap_{1\leq p<\infty} L^p[0,1]$ equipped with the topology of the L^p-norms, $1 \leq p < \infty$ (G.R. Allan) and the algebra $M[0,1]$ of all measurable functions on $[0,1]$ (modulo equality a.e.), endowed with the topology of convergence in measure, which is not necessarily locally convex (P.G. Dixon).

To our knowledge, up to now, there is nothing in the literature about tensor products of GB^*-algebras. So, this talk is devoted to the investigation of this subject matter. First, we can show that if X is a Hausdorff locally compact space and A a C^*-like locally convex *-algebra (with continuous multiplication), then the complete locally convex $*$-algebra of all A-valued continuous functions on X, $C(X,A) \cong C_c(X)\widehat{\otimes}A$, is a GB^*-algebra, under the injective tensorial topology (note that "c" denotes the topology of compact convergence on the algebra $C(X)$ of all $\mathbb{C}$-valued continuous functions on X). Furthermore, sufficient and necessary conditions will be given, such that the complteted tensor product of two GB^*-algebras under a "$*$-admissible" topology is again a GB^*-algebra. Applications concerning (unbounded) $*$-representation theory of tensor product GB^*-algebras will be presented.

The talk is based on a joint work with A. Inoue and M. Weigt.