Optimal H^2 solutions to a rational Bezout type equation

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Let G be a stable rational $m \times p$ matrix function where $m < p$. In particular, G is a rational function in $H^2_{m,p}$. We study Bezout type equations of the form

$$G(z)X(z) = I_m, \quad z \in \mathbb{D},$$

where the solution X is a rational function in $H^2_{p,m}$. Given a stable state space realization of G, we present necessary and sufficient conditions for $G(z)X(z) = I_m$ to be solvable in terms of solutions of associated Riccati and Stein equations. In this case, a state space formula is given for the optimal H^2 solution. Another state space formula for a special inner function yields the set of all solutions. A state space space solution for the Tolokonnikov completion appearing in the rational corona problem is also given. The proofs use operator theory techniques. Finally, numerical examples with applications to systems theory are presented.

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