Oka's principle in the Fredholm theory for Frechet algebras of Fourier operators

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The classical result of the homotopy transition from continuous to holomorphic maps (Oka) from holomorphy regions to Fredholm operators on Banach spaces (1971, 1984) is extended in several directions. We include in the theory the symmetric Hrmander class (1,1) of pseudodifferential operators. This class is known to be not spectrally invariant; it has not an open group of invertible elements; but using commutator methods involving hard analysis the submultiplicativity of this class(1,1) can be shown. This is the key to apply infinite products and meromorphic decompositions. The Arens- Royden theorem, a version of the Oka principle for Banach algebras, is extended for maps with values in various sets of Fredholm operators. The fundamental contributions of Grauert (1957) and Gromov (1989) to the Oka principle had an essential revival during the last decade by the work of Forstneric et al. and Lempert et al. (see e.g. Notices AmS,vol.57 2010,p. 50- 52 and Ann. Sc. Ec. Norm. Sup, vol. 40, 2007). Also the book of I. Gohberg and J. Leiterer (Birkhuser 2009) is an excellent starting point for the extension of the Oka principle to maps with values in the set of Fredholm operators. These contributions lead to a series of challenging problems also for operators in Hilbert spaces with maps from infinite dimensional holomorphy regions into Oka manifolds (see above Notices AMS). The operator methods can be applied to treat microlocal Frchet algebras on ramified or stratified manifolds using Lie group representations and commutator procedures with vector fields. For this purpose the Leibniz - Nelson product rule for closed operators and there commutators is analysed with an appropriate differentiability producing submultiplicative norms. There are relations to the dissertation of J. Ditsche and the work of M. Denz (Mainz 2007, 2008).