Reduced resolvent formula and weak operator equations, a bridge between block operator matrices and numerical linear algebra

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We study the stability of eigenvalues and eigenvectors of self-adjoint operators which are defined by quadratic forms under a large class of singular perturbations. We divide the spectrum of an operator into the target component and the unwanted component of the spectrum. This dichotomy induces a natural block operator matrix representation of the associated resolvent. The associated spectral projections form a partition of unity. Furthermore, a similar block operator is induced by any partition of unity such that the intersection of the range of projection and the form domain of the operator is dense. We study such resolvent-s using basic matrix factorizations of numerical linear algebra and show how in an infinite dimensional setting the nonexistence of a factorization can be used to characterize a certain operator theoretic phenomenon. We revisit the use of techniques such as Schur complements, operator Riccati and Sylvester equations to quantitatively study the perturbation theory of spectra of operators defined as quadratic forms. Some applications will be presented which include both a study of the asymptotic exactness of error estimators employed in modern adaptive finite element approximations as well as asymptotic study of spectral properties in the limit of large penalty (e.g. as employed in the study of Maxwell or Stokes eigenvalue problems). This is a joint work with V. Kostrykin, K. Makarov, J. Ovall and K. Veselic.