# Besov class calculi for discrete and continuous operator semigroups

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It is well-known that the class of power-bounded operators on a Hilbert space is less well-behaved than the class of contractions. For example, for a contraction $T$ on a Hilbert space $H$ von Neumann's inequality states that one can estimate

$$\|p(T)\|_{\mathcal{L}(H)} \leq \|p\|_{H^\infty(\mathbb{D})}$$

for every polynomial $p \in \mathbb{C}[z]$, but even the weaker inequality

$$\|p(T)\|_{\mathcal{L}(H)} \lesssim \|p\|_{H^\infty(\mathbb{D})}$$

fails for a general power-bounded operator. Peller (1982) has proved that for a power-bounded operator $T$ on a Hilbert space one can estimate

$$\|p(T)\|_{\mathcal{L}(H)} \lesssim \|p\|_{B^0_{\infty,1}} := \sum_{n\geq 0} \|\varphi_n * p\|_{H^\infty(\mathbb{D})},$$

where $(\widehat{\varphi_n})_{n\geq 0}$ is a dyadic partition of unity of $\mathbb{Z}_+$, and $*$ is convolution on $\mathbb{T}$. This Besov class estimate is considerably stronger than the obvious estimate $\|p(T)\|_{\mathcal{L}(X)} \lesssim \|\widehat{p}\|_{\ell^1}$, best possible in the class of all power-bounded operators on Banach spaces.

Based on recently discovered *transference principles* for semigroups we present a completely different proof of Peller's result and establish several generalizations. First, we show that an analogous theorem holds for $C_0$-semigroups on Hilbert spaces. Second we establish general Banach space versions of these results involving the concept of $\gamma$-radonifying operators, introduced by Kalton and Weis in 2004. Finally, we sketch generalizations to operators/semigroups on $L_p$-spaces or, more general, UMD Banach spaces. In particular, it follows from our results that the singular integral

$$\mathrm{PV}-\int_0^{t+1} \frac{T(s)x}{s-t}\, ds \qquad (x \in X,\, t > 0).$$

converges whenever $(T(s))_{s\geq 0}$ is a $C_0$-semigroup on a UMD Banach space.

The talk is based on our paper [Math. Ann. 345 (2) (2009), 245-265] and the recent preprint *Transference Principles for Semigroups and a Theorem of Peller*.