Convex Matrix Inequalities vs Linear Matrix Inequalities

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A substantial advance in optimization starting in the 1990's was the realization that problems in many areas, like linear system control, combinatorics, statistics convert directly to matrix inequalities, abbreviated MIs, of which by dent of great cleverness some convert to Linear Matrix Inequalities, LMI's. A basic question is: which Matrix Inequalities are in fact Linear Matrix Inequalities? Clearly, LMIs are convex, but what about the converse?

How much more restricted are LMIs than Convex MIs?

There is getting to be a reasonable road map to this problem with much left to be proved. It involves use and development of techniques from areas like functional analysis, real algebraic geometry (polynomial inequalities) and matrix theory. In this talk we give results and conjectures on the answer to the LMI vs convexity question.

A further direction involves matrix variables and is the issue of transforming problems to convex ones. This leads to noncommutative versions of classical theorems in several complex variables.