Compact Toeplitz operators for weighted Bergman spaces on bounded symmetric domains

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It is well known that the Berezin transform $\tilde{g}^t$ of the Toeplitz operator T_g^t acting on the Segal-Bargmann space of square integrable entire functions with respect to a time dependent Gaussian measure, is the solution of the heat equation at time $t > 0$ with initial data g. As it was recently shown, if g is a function on $\mathbb{C}^n$ with a bounded mean oscillation, $\tilde{g}^{t_0}$ vanishes at infinity for **some fixed** time t_0 if and only if $\tilde{g}^t$ vanishes at infinity for **any** time $t > 0$.

If we replace $\mathbb{C}^n$ by a bounded symmetric domain Ω , we can still define the weighted Berezin transform $\tilde{g}_\nu$ of the Toeplitz operator T_g^ν acting on the weighted Bergman space of Ω. This Berezin transform generalizes the heat transform on $\mathbb{C}^n$ where the time parameter is replaced by the weight parameter. In this talk we consider $\Omega \subset \mathbb{C}^n$ of type (r, a, b) in its Harish-Chandra realization. Under some conditions on the weights ν and ν_0 we show the existance of $C(\nu, \nu_0) > 0$, such that

$$\|\tilde{g}_{\nu_0}\|_\infty \leq C(\nu, \nu_0) \left\|T_g^\nu\right\|_\nu$$

for all g in a suitable class of symbols containing $L^\infty(\Omega)$. As a consequence we prove that the compactness of T_g^ν (or the vanishing of $\tilde{g}_\nu$ near the boundary of Ω) is **independent** of the weight ν, whenever $g \in L^\infty(\Omega)$ and $\nu > C$ where C is a constant depending on (r, a, b).