

# Noncommutative Functions: Algebraic and Analytic Results

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Functions on  $d$ -tuples of square matrices of all sizes which take values on square matrices of same sizes and respect direct sums and joint similarities of matrices are called noncommutative functions over  $\mathbb{C}^d$ . A list of examples includes, but is not limited to, noncommutative polynomials or formal power series, and noncommutative rational expressions. A natural setting for the notion of a noncommutative function over a Banach space is provided in the context of operator spaces. Noncommutative functions arise in various areas of mathematics and its applications: noncommutative algebra, operator theory, free probability, noncommutative multidimensional system theory, robust control, optimization, etc. In this talk, a difference-differential calculus for noncommutative functions will be introduced, including a noncommutative version of the Taylor formula. Some algebraic and analytic applications of this calculus will be presented.

The talk is based on a joint work with V. Vinnikov.