Stieltjes functions and stable entire functions

V. Katsnelson

he function $\psi(z)$ belongs to the class $\boldsymbol{S}$ if ψ is holomorphic in the domain $\mathbb{C} \setminus (-\infty, 0]$ and satisfy the conditions:

1. $\psi(x) \geq 0$ for $x > 0$; **2**. $\operatorname{Im} \psi(z) \leq 0$ for $\operatorname{Im} z \geq 0$.

It is known that a function ψ belongs to the class $\boldsymbol{S}$ if and only if ψ admits the representation

$$\psi(z) = c + \int\limits_{-\infty}^{0} \frac{d\sigma(\lambda)}{\lambda + z},$$

where $d\sigma$ is a non-negative measure satisfying the condition $\int\limits_{-\infty}^{0} \frac{d\sigma(\lambda)}{1+|\lambda|} < \infty$ and c is a non-negative constant.

If $\{a_j\}$, $\{b_j\}$ are two sequences of non-negative numbers which interlace: $0 \leq a_1 < b_1 < a_2 < b_2 < \ \ldots$, then the product

$$\psi(z) = \prod_{j=1}^{\infty} \frac{z + a_j}{z + b_j}$$

converges for $z \in \mathbb{C} \setminus (-\infty, 0)$ and defines a function of the class $\boldsymbol{S}$.

The following result is obtained:

If ψ is a function of the class $\boldsymbol{S}$, $\psi(+0) < \infty$, and

$$f(z) = \sum_{k=0}^{\infty} \psi(k) \frac{z^k}{k!},$$

then f is a stable entire function of exponential type. In particular, all roots of f lie in the open left half plane.

This result allows to construct some new classes of meromorphic Laguerre multiplier sequences.

(The notion of meromorphic Laguerre multiplier sequence was introduced and discussed by T. Craven and G. Csordas in J. Math. Anal. Appl., **314** (2006), 109-125.)