Completeness of translates of entire functions and hypercyclic operators

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Let $H(\mathbb{C})$ be the space of all entire functions with uniform convergence topology. For a wide class of entire functions (including, for example, Airy functions) we prove that the systems of their translates are complete in $H(\mathbb{C})$. As a corollary we obtain new classes of hypercyclic operators on $H(\mathbb{C})$.

Consider the translation operator on $H(\mathbb{C})$: $f(z) \to S_\lambda f(z) \equiv f(z+\lambda)$. A well-known theorem of Godefroy and Shapiro [1] states that every continuous linear operator T on $H(\mathbb{C})$, satisfying $TS_\lambda - S_\lambda T = 0$ is hypercyclic, if it is not a scalar multiple of the identity. We obtain the following more general result:

Theorem 1. Let $T : H(\mathbb{C}) \to H(\mathbb{C})$ be a continuous linear operator such that 1)$\exists a \in \mathbb{C} : TS_\lambda f - S_\lambda T f = a\lambda S_\lambda f$, $\forall \lambda \in \mathbb{C}$, $\forall f \in H(\mathbb{C})$; 2) $\ker T \neq \{0\}$, $\ker T \neq H(\mathbb{C})$. Then T is hypercyclic.

This result is based on the following theorem.

Theorem 2. Let $T : H(\mathbb{C}) \to H(\mathbb{C})$ be a continuous linear operator such that $TS_\lambda f - S_\lambda T f = \lambda S_\lambda f$, $\forall \lambda \in \mathbb{C}$, $\forall f \in H(\mathbb{C})$. Then the system $\{S_\lambda g, \lambda \in \Lambda \subset \mathbb{C}\}$ is complete in $H(\mathbb{C})$, if $g \in \ker T \setminus \{0\}$ and Λ contains a limit point.

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1. Godefroy G., Shapiro J. H. // J. Funct. Anal. 1991, 98:2, 229–269.