Matrix-valued truncated K-moment problems in several variables

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In this presentation, the matrix-valued truncated K-moment problem on $\mathbb{R}^d$ and $\mathbb{C}^d$ will be considered. The matrix-valued truncated K-moment problem on $\mathbb{R}^d$ requires necessary and sufficient conditions for a multisequence of Hermitian matrices $\{S_\gamma\}_{\gamma\in\Gamma}$, where Γ is a finite subset of $\mathbb{N}_0^d$, to be the corresponding moments of a positive matrix-valued Borel measure σ and also the support of σ must lie in some given non-empty set $K \subseteq \mathbb{R}^d$, i.e.

$$S_\gamma = \int_{\mathbb{R}^d} \xi^\gamma d\sigma(\xi), \quad \gamma \in \Gamma, \tag{1}$$

and

$$\operatorname{supp} \sigma \subseteq K. \tag{2}$$

In a joint work with Hugo J. Woerdeman, given a non-empty set $K \subseteq \mathbb{R}^d$ and a finite multisequence, indexed by a certain family of finite subsets of $\mathbb{N}_0^d$, of Hermitian matrices we obtain necessary and sufficient conditions for the existence of a finitely atomic measure which satisfies (1) and (2). In particular, our result can handle the case when the indexing set that corresponds to the powers of total degree at most $2n+1$. We will also discuss a similar result in the complex setting.