Krein signature and eigencurves of non-conservative gyroscopic systems

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Frequency loci crossing and veering phenomena are closely related to wave propagation and instabilities in fluids and structures. In engineering applications the crossings of the eigencurves are typically observed in gyroscopic or potential systems in the presence of symmetries, such as rotational or spherical one. The examples are perfect bodies of revolution that serve for modeling turbine wheels, disk and drum brakes, tires, clutches, vibrating gyroscopes, paper calenders and other rotating machinery. The frequency-rotational speed plot, or Campbell diagram, of such gyroscopically coupled systems consists of eigencurves that intersect each other, representing the frequencies of forward-, backward-, and reflected traveling waves. The intersections correspond to the double eigenfrequencies, or doublets. Zero doublets occur at the critical speeds of rotation corresponding to a stationary backward traveling wave in the non-rotating frame. It is well-known that the stationary conservative loads induce divergence and flutter instabilities either at the critical speeds or at the non-zero doublets above the critical speeds (mass and stiffness instabilities). In the vicinity of such doublets rings of complex eigenvalues originate (bubbles of instability). The same loads, however, only veer the eigencurves away near the crossings situated below the lowest critical speed without destabilization. MacKay (1986) explains this phenomenon using the notion of symplectic (Krein) signature of eigenvalues. He shows that the unfolding of the doublets due to Hamiltonian perturbations is represented by the conical eigenvalue surfaces of two types. Their one-dimensional slices fully describe all possible frequency loci veering scenarios in conservative gyroscopic systems. When damping and non-conservative forces act on a gyroscopically coupled system with symmetry, the frequency veering scenarios become more complicated. Both frequencies and growth rate loci can cross and avoid crossing. Moreover, flutter instabilities can occur below the first critical speed. In the present talk I show that as in the case of a conservative gyroscopic system, all possible frequency and growth rate loci veering and crossing scenarios in the presence of damping and non-conservative positional forces are described by means of the one-dimensional slices of singular eigenvalue surfaces of only two types. These surfaces are different from the MacKay's cones; they have two singular points locally equivalent to the Whitney umbrella. The type of the singular surface into which the MacKay's cones unfold under non-Hamiltonian perturbation is sharply determined by the symplectic (Krein) signature of the double eigenvalue of the unperturbed gyroscopic system. The singularities found, connect the problems of wave propagation in the rotating continua with that of electromagnetic and acoustic wave propagation in non-rotating anisotropic chiral media. As examples, eigencurves in a model of a rotating shaft under non-conservative loading and in a non-self-adjoint boundary value problem for a rotating circular string passing through the eyelet with friction are studied in detail.