Covariant spectrum and Krein spaces

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The group $G = SL_2(\mathbb{R})$ consists of 2×2 matrices with real entries and the unit determinant. Let H be a Hilbert space, for a bounded linear operator T on H and a vector $v \in H$ we define the function on $SL_2(\mathbb{R})$ by

$$c(g) = (cT + dI)^{-1}v \qquad \text{where} \quad g^{-1} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL_2(\mathbb{R}).$$

Consider the space $L(G, H)$ of H-valued functions spanned by all left translations of $c(g)$. The left action of $SL_2(\mathbb{R})$ on this space is a linear representation of this group.

A covariant functional calculus is defined as a continuous linear intertwining map between two representations of a group: the first representation acts on scalar-valued functions, the second—on vector valued ones. The intertwining property replaces an algebra homomorphism in the standard construction of functional calculus. The above action of $SL_2(\mathbb{R})$ is an example of covariant calculus.

A covariant spectrum is defined as the support of covariant functional calculus, that is a decomposition of the intertwining map into primary subrepresentations. Such components associated with the above $SL_2(\mathbb{R})$ calculus fall into three large classes, with the pairings defined by the usual and indefinite inner products as well as degenerate one.

We discuss some aspects of covariant spectral theory and its connections with characteristic functions, the functional model and Krein spaces.