Direct and inverse problems for discrete Schrödinger operators on $Z^d, d \geq 2$

E. Korotyaev

We consider the discrete Schrödinger operator $H = \Delta + V$ on $Z^d, d \geq 2$ with real potential $V(n) = O(|n|^{-a}), a > d$ as $|n| = |n_1| + ... + |n_d| \to \infty$, where $n = (n_j)_1^d \in Z^d$. The following results are obtained:

1) Eigenvalues. We show the operator has finite number of eigenvalues including multiplicity. Moreover, the eigenvalues on the spectrum are absent, similar to the Kato result for the continuous case.

2) We solve few inverse problems. In particular, the S-matrix determines the potential uniquely, and we present the algorithm to recover the potential.

3) We study resonances for the operator H with finitely supported potential. We show that the Riemann surface for the operator has infinitely many sheets for any dim $d \geq 2$. Some results about the distribution of resonances are obtained. Recall that in the case $d = 1$ (and in the continuous case for any dimension) the Riemann surface for the Schrödinger operator has two sheets.

4) We determine the trace formulas for H.

The talk is based on a joint work with H. Isozaki.