The Hörmander functional calculus

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Let $d \in \mathbb{N}$ and $p \in (1, \infty)$. A theorem of Hörmander says that if $f : [0, \infty) \to \mathbb{C}$ satisfies

$$\sup_{r>0} \int_r^{2r} |s^k f^{(k)}(s)|^2 \frac{ds}{s} < \infty \quad (k = 0, 1, \ldots \lfloor \frac{d}{2} \rfloor + 1)$$

then f is a radial Fourier multiplier on $L^p(\mathbb{R}^d)$, i.e. the mapping

$$L^p(\mathbb{R}^d) \to L^p(\mathbb{R}^d),\, g \mapsto f(-\Delta)g = [f(|\cdot|^2)\hat{g}]\check{}$$

is bounded.

We put this into a more general framework. Consider a generator A of a semigroup with spectrum contained in $[0, \infty)$. We compare conditions on the semigroup with multiplier theorems modeled after Hörmander's one above.