Vanishing of the Lyapunov exponent

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A Jacobi operator is bounded linear operator on $\ell 2(\mathbb{N})$ acting by

$$Hu(n) = a(n+1)u(n+1) + b(n)u(n) + a(n)u(n-1),$$

where $u(0) = 0$ and $a(n)$, $b(n)$ are bounded real valued sequences. We will also assume that $a(n) > 0$ is bounded away from zero.

The Lyapunov Exponent $L(E)$ is given by the maximal exponential growth of solutions of $Hu = Eu$ ignoring the boundary condition at 0. I will describe consequences of the Lyapunov exponent vanishing on the essential spectrum of H.

The most basic result is that if the essential spectrum is $[-2, 2]$ and the Lyapunov exponent vanishes on it, then $a(n) - 1$ and $b(n)$ Cesàro sum to zero.