On a class of J-self-adjoint operators with empty resolvent set

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In contrast to self-adjoint operators in Hilbert spaces (which necessarily have a purely real spectrum), self-adjoint operators A in Krein spaces $(\mathfrak{H}, [\cdot,\cdot]_J)$ (J-self-adjoint operators) may have spectra which are only symmetric with respect to the real axis. Moreover, the situation where $\sigma(A) = \mathbb{C}$ (i.e., A has the empty resolvent set) is also possible. It is natural to suppose that the relation $\sigma(A) = \mathbb{C}$ indicates some special structure of a J-self-adjoint operator A. We illustrate such a point by considering the set Σ_J of all J-self-adjoint extensions A of the symmetric operator S with deficiency indices $< 2, 2 >$ which commutes with J. In that case the existence of at least one $A \in \Sigma_J$ with empty resolvent set is equivalent to the existence of an additional fundamental symmetry R in $\mathfrak{H}$ such that

$$SR = RS, \qquad JR = -RJ.$$

The operators J and R can be interpreted as basis (generating) elements of the complex Clifford algebra $\mathcal{C}l_2(J, R) := \operatorname{span}\{I, J, R, JR\}$ and they give rise to a 'rich' family of exactly solvable models of $\mathcal{PT}$-symmetric quantum mechanics (PTQM) explaining (at an abstract level) the appearance of exceptional points on the boundary of the domain of the exact $\mathcal{PT}$-symmetry in PTQM.

The talk is based on joint works with C. Trunk and with S. Albeverio and U. Günther.