Convex cones of generalized positive rational functions and Nevanlinna-Pick interpolation

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As a motivation recall that the Nevanlinna-Pick interpolation problem is to find for given vectors $x, y \in \mathbb{C}^n$ a (complex scalar) rational function f, from a certain class so that $y_j = f(x_j)$ $j = 1, \ \ldots\ n$. All solutions may be parameterized by the corresponding Pick matrix Π. Specifically, in the framework of Schur functions, mapping the unit disk to itself, $[\Pi]_{jk} = \frac{1-y_j^* y_k}{1-x_j^* x_k}$; and in the framework of positive functions, $\mathcal{P}$, mapping the right half plane to itself, $[\Pi]_{jk} = \frac{y_j^*+y_k}{x_j^*+x_k}$.

The analogy is more intricate when one addresses Nevanlinna-Pick interpolation problem over generalized Schur functions, mapping the unit circle to the unit disk and Generalized Positive functions, $\mathcal{GP}$, mapping the imaginary axis to the right half plane.

The key to our approach is following factorization result,

$$\psi \in \mathcal{GP} \iff \psi(s) = g(s)p(s)g(s)^{\#} \ , \quad p \in \mathcal{P},$$

with g, g^{-1} analytic in the left half plane and $g(s)^{\#} := g(-s^*)^*$. This induces a convex partitioning of all $\mathcal{GP}$ functions into sets of function sharing the same poles and zeroes in the right half plane. Within these sets, we introduce a simple scheme for Nevanlinna-Pick interpolation.

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