Spectral asymptotic of elliptic operators with non-local boundary conditions

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Recently the boundary triplets approach to the parametrisation of self-adjoint realisations of some elliptic expression on a smooth domain was suggested. There are two main ways. The first way is based on the ordinary triplet and allows to parametrise all self-adjoint realisations. The second way is to use the quasi boundary triplet or something similar. In such a case we parametrise only realisations with the certain regularity of the operator domain. The advantage of this approach that one can take the Dirichlet trace and the Neumann trace as the boundary mappings. The Weyl function becomes the Dirichlet-to-Neumann or the Neumann-to-Dirichlet map. These objects are very natural in the Analysis of PDE.

Using quasi boundary triples we prove various Schatten-von Neumann estimates of resolvent differences of two self-adjoint (max. dissipative or max. accumulative) realisations of some second-order elliptic expression on a bounded (or unbounded) domain with a compact smooth boundary. Among these estimates there are some of the well-known estimates due to M. Sh. Birman, M. Z. Solomyak, G. Grubb proved in a simpler way and some new. In particular the resolvent difference of the classical Robin and Neumann realisations has better asymptotic of singular numbers than the resolvent difference of Dirichlet and Neumann realisations. We also include estimates for differences of integer powers of resolvents.

These estimates together with variational principle of T. Kato allow us to compare generalised spectra of two realisations.

This is a joint work with J. Behrndt and M. Langer.