Positive decompositions of selfadjoint operators

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Given a linear bounded selfadjoint operator a on a complex separable Hilbert space $(\mathcal{H}, \langle ., . \rangle)$, we study the decompositions of a as a difference of two positive operators whose ranges satisfy an angle condition. These decompositions are related to the canonical decompositions of the indefinite metric space $(\mathcal{H}, \langle a., . \rangle)$, associated to a. As an application, we characterize the orbit of congruence of a, $O_a = \{gag^* : g \in GL(\mathcal{H})\}$, in terms of its positive decompositions. When a has closed range, we show that O_a with a suitable metric has a differentiable structure, in fact, it is a principal fibre bundle.

The talk is based on a joint work with G. Fongi.