Perturbation determinants and trace formulas for non-additive perturbations

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A classical notion of the perturbation determinant (PD) is naturally defined for a pair of closed operators T_0 and T with common domains of definition, $\mathrm{dom}\,(T) = \mathrm{dom}\,(T_0)$. Namely, if $V(T_0 - z)^{-1} \in S_1$ (the trace class ideal), and $V := T - T_0$, then the PD is defined by

$$D(z) = det[I + V(T_0 - z)^{-1}].$$

We consider a pair $\{T, T_0\}$ in the case $\mathrm{dom}\,(T) \neq \mathrm{dom}\,(T_0)$ assuming only that

$$(T - z)^{-1} - (T_0 - z)^{-1} \in S_1.$$

We discuss a notion of PD in the framework of extensions theory of symmetric operators by applying boundary triplet approach. We compute the PD by means of the Weyl function and the corresponding boundary operators. We obtain different trace formulas and complete some results of M.G. Krein, V. Adamyan, B. Pavlov and others.

The talk is based on the joint work with H. Neidhardt.