Basisness results for some singular pencils of differential operators arising in hydrodynamics

M. Marletta

Many classical linear stability problems in hydrodynamics, MHD and other areas of physics give rise to operator pencil problems. Since the early 1990s, Trefethen has pointed out some of the dangers of this approach which arise due to pseudospectral phenomena.

However there are some more fundamental issues which afflict this methodology. Benilov, O'Brien and Sazonov proposed a model for drop formation in a rotating fluid in which the eigenfunctions form a complete set but linear stability fails even though all the eigenvalues lie in the stable half-plane. The reason for this failure is that the eigenfunctions (there are no associated functions) do not form a Riesz basis in an appropriate space.

Basisness results are rarely discussed in the fluid dynamics literature despite their central importance for linear stability, for the simple reason that they are generally rather difficult to prove. The first basisness results for the classical Orr-Sommerfeld problem, for instance, only date back to the 1980s.

In this talk I shall review some older work on pencil problems with λ-dependent boundary conditions, as well as some more recent work on singular pencil problems of a type which arise in, e.g., the analysis of pipe Poiseuille flow. For the problems with λ-dependent boundary conditions some lack-of-basisness results will be shown, while for the singular problems with λ-independent boundary conditions basisness results are obtained subject to a completeness hypothesis.

This talk is based on joint work with A. Shkalikov and Ch. Tretter.