Decay estimates for singular extensions of vector-valued Laplace transforms

M.M. Martínez

Let X be a Banach space and let $f : [0, \infty) \to X$ be a bounded measurable function. The Laplace transform of f is given by the Bochner integral

$$\hat{f}(z) := \int_0^\infty f(t)e^{-zt}dt\,, \quad \Re z > 0.$$

The use of vector-valued Laplace transform techniques has led to an important advance in the theory of linear evolution equations and semigroup theory. Among these techniques, ones of the most challenging are those concerning to the asymptotic behaviour and, in particular, *Tauberian theorems*, in which asymptotic properties of a function are deduced from properties of its transform.

We shall consider a function $f \in L^\infty([0, \infty); X)$ whose Laplace transform extends analytically to some region containing $i\mathbb{R} \setminus \{0\}$, possibly having a pole at the origin. The aim of this talk is to present estimates of the decay of certain slight modification of f in terms of the growth of $\hat{f}$ along the imaginary axis (up to the origin).

From this theorem, we will deduce some new decay estimates for C_0-semigroups of operators. Let $(T(t))_{t\geq 0}$ be a bounded C_0-semigroup on a Banach space whose infinitesimal generator A is such that $\sigma(A) \cap i\mathbb{R} \subseteq \{0\}$, where $\sigma(A)$ denotes the spectrum of A. We then estimate the decay at infinity of $\|T(t)A(1-A)^{-2}\|$ in terms of the growth of the resolvent operator along the imaginary axis. The same technique is applied to get similar decay estimates for bounded C_0-semigroups whose infinitesimal generator has an arbitrary finite number of spectral values on the imaginary axis. These results are in the spirit of the ones recently obtained by C. J. K. Batty and T. Duckaerts and they are motivated by applications to wave equations.