Operators with specification properties

F. Martínez-Giménez

A continuous map $f : X \to X$ on a compact metric space (X, d) has the strong specification property (SSP) if for any $\delta > 0$ there is a positive integer N_δ such that for any integer $s \geq 2$, any set $\{y_1, \dots, y_s\} \subset X$ and any integers $0 = j_1 \leq k_1 < j_2 \leq k_2 < \cdots < j_s \leq k_s$ satisfying $j_{r+1} - k_r \geq N_\delta$ for $r = 1, \dots, s-1$, there is a point $x \in X$ such that, for each positive integer $r \leq s$ and all integers i with $j_r \leq i \leq k_r$, the following conditions hold:

$$d(f^i(x), f^i(y_r)) < \delta,$$

$$f^n(x) = x, \quad \text{where } n = N_\delta + k_s.$$

The special case $s = 2$ is called periodic specification property (PSP).

We are interested in the study of continuous and bounded operators on Banach spaces having specification properties. Based on concrete examples, it seems that the periodic specification property is stronger than chaos in the sense of Devaney but a complete study of this property for bounded operators must be done. This include the relationships with other well known dynamical properties for operators like hypercyclicity, mixing, different notions of chaos, etc.

We have made some advances in this topic and we found characterizations for different types of specification properties for backward shift operators on Banach sequence spaces. Namely, we proved that for a bounded shift operator B defined on $\ell^p(v)$, $1 \leq p < \infty$ (or $c_0(v)$) the following conditions are equivalent:

(i) $\sum_{n=1}^{\infty} v_n < \infty$ ($\lim_{n\to\infty} v_n = 0$).

(ii) B has SSP.

(iii) B has PSP.

(iv) B is Devaney chaotic.

Joint work with S. Bartoll and A. Peris.