Variational principles for eigenvalues of operator functions and block operator matrices

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In this talk functions T are considered which are defined on some interval I and whose values are (in general unbounded) self-adjoint operators in a Hilbert space. A point $\lambda \in I$ is called an eigenvalue of T if there exists a vector $x \in \operatorname{dom}(T(\lambda))$, $x \neq 0$, so that $T(\lambda)x = 0$. Under very mild assumptions a variational inequality for eigenvalues of T is proved, which can also be used to show that a certain subinterval of I is free of spectrum. In various situations this inequality turns out to be an equality, which is not true in general. The general variational inequality and equality are applied to the Schur complement of a block operator matrix, which yields estimates for the spectrum and eigenvalues of this block operator matrix in certain gaps of the essential spectrum.

The talk is based on joint work with M. Strauss.