# Smoothness of Hill's potential and lengths of spectral gaps

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Let $\{\gamma_q(n)\}_{n\in\mathbb{N}}$ be the lengths of spectral gaps in a continuous spectrum of the Hill-Schrödinger operators

$$S(q)u = -u'' + q(x)u, \quad x \in \mathbb{R},$$

with 1-periodic real-valued potentials $q \in L^2(\mathbb{T})$, $\mathbb{T} := \mathbb{R}/\mathbb{Z}$.

Let us consider the map

$$\gamma : \ q \mapsto \{\gamma_q(n)\}_{n\in\mathbb{N}}.$$

Then due to Garnett & Trubowitz, *Comm. Math. Helv.*, **59** (1984) we have

$$\gamma(L^2(\mathbb{T})) = l^2_+(\mathbb{N}), \quad l^2_+(\mathbb{N}) := \left\{\{a(k)\}_{k\in\mathbb{N}} \in l^2(\mathbb{N}) \mid a(k) \geq 0, \ k \in \mathbb{N}\right\}.$$

**Theorem 1 ([1]).** *Let weight function $\omega : [1,\infty) \to (0,\infty)$. We prove that under the condition*

$$\exists s \in [0,\infty) : \quad k^s \ll \omega(k) \ll k^{s+1}, \ k \in \mathbb{N},$$

*the map $\gamma : q \mapsto \{\gamma_q(n)\}_{n\in\mathbb{N}}$ satisfies the equalities*:

$$i) \quad \gamma\left(H^\omega(\mathbb{T})\right) = h^\omega_+(\mathbb{N}), \qquad ii) \quad \gamma^{-1}\left(h^\omega_+(\mathbb{N})\right) = H^\omega(\mathbb{T}).$$

Here,

$$f(x) = \sum_{k\in\mathbb{Z}} \widehat{f}(k)e^{ik\pi x} \in H^\omega(\mathbb{T}) \Leftrightarrow \sum_{k\in\mathbb{Z}} \omega^2(k)|\widehat{f}(k)|^2 < \infty,$$

$$a = \{a(k)\}_{k\in\mathbb{N}} \in h^\omega_+(\mathbb{N}) \Leftrightarrow \sum_{k\in\mathbb{N}} \omega^2(k)|a(k)|^2 < \infty, \quad a(k) \geq 0, \ k \in \mathbb{N}.$$

All results were obtained jointly with V. Mikhailets.

[1] Mikhailets, V., Molyboga, V., *Smoothness of Hill's potential and lengths of spectral gaps*, arXiv: math.SP/1003.5000.