Bounds on variation of spectral subspaces under J-self-adjoint perturbations

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Given a self-adjoint involution J on a Hilbert space $\mathfrak{H}$, we consider a J-self-adjoint operator L on $\mathfrak{H}$ of the form $L = A + V$ where A is a possibly unbounded self-adjoint operator commuting with J and V a bounded J-self-adjoint operator anti-commuting with J. We establish optimal estimates on the position of the spectrum of L with respect to the spectrum of A and we obtain norm bounds on the operator angles between maximal uniformly definite reducing (and, in particular, spectral) subspaces of the perturbed operator L and those of the unperturbed operator A. All the bounds are given in terms of the norm of V and the distances between pairs of disjoint spectral sets associated with the operator A or/and the operator L. As an example, the quantum harmonic oscillator under a $\mathcal{PT}$-symmetric perturbation is discussed. The a priori and a posteriori sharp norm bounds obtained for the operator angles generalize the celebrated Davis-Kahan trigonometric theorems to the case of J-self-adjoint perturbations.

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