Maximal regularity of cylindrical parameter-elliptic boundary value problems

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For parameter-elliptic boundary value problems in domains $V \subset \mathbb{R}^k$ with compact boundary, $\mathcal{R}$-sectoriality of the related L^p-realizations is known. We make use of this result to show $\mathcal{R}$-sectoriality for the L^p-realizations of a class of boundary value problems in cylindrical domains $W \times V$, where W is either given as the fullspace $\mathbb{R}^n$ or the cube $[0, 2\pi]^n$. Due to a result of L. Weis, this gives maximal regularity for the corresponding Cauchy problem. The differential operators A under consideration are assumed to resolve into two parts $A = A_1 + A_2$, both parameter-elliptic, such that A_1 acts merely on W and A_2 acts merely on V. As a strong tool to treat model problems of this kind, continuous and discrete operator-valued Fourier multiplier theorems are used. In this context, $\mathcal{R}$-boundedness of operator families, namely the range of $(\lambda + a_1(\cdot) + A_2)^{-1}$ plays a major role. This indicates how spectral properties of A_2 can be used to derive according properties for A. To some extent this approach allows to treat vector-valued boundary value problems in the above domains. In the case where W is given as a cube, periodic and antiperiodic boundary conditions are treated.

The talk includes a joint work with J. Saal.