On the unitary equivalence of absolutely continuous parts of self-adjoint extensions

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The classical Weyl-von Neumann theorem states that for any self-adjoint operator A_0 in a separable Hilbert space $\mathfrak{H}$ there exists a (non-unique) Hilbert-Schmidt operator $C = C^*$ such that the perturbed operator $A_0 + C$ has purely point spectrum. We are interesting whether this result remains valid for non-additive perturbations by considering the set Ext_A of self-adjoint extensions of a given densely defined symmetric operator A in $\mathfrak{H}$ and some fixed $A_0 = A_0^* \in \mathrm{Ext}_A$. We show that for some A and A_0 the absolutely continuous spectrum $\sigma_{ac}(A_0)$ of A_0 remains stable under compact non-additive perturbations, that is, $\sigma_{ac}(A_0) = \sigma_{ac}(\widetilde{A})$ for any $\widetilde{A} \in \mathrm{Ext}_A$ provided that the resolvent difference $(\widetilde{A} - i)^{-1} - (A_0 - i)^{-1}$ is compact and $\sigma_{ac}(A_0)$ might only increase if not. We investigate the same property for the absolutely continuous part of A_0 and show that for a wide class of symmetric operators A the absolutely continuous parts of $\widetilde{A} \in \mathrm{Ext}_A$ and A_0 are unitarily equivalent whenever their resolvent difference is compact. Namely, it is true whenever the Weyl function $M(\cdot)$ of a pair $\{A, A_0\}$ has bounded limits $M(t) := w{-}\lim_{y\to+0} M(t+iy)$ for a.e. $t \in \mathbb{R}$. This result is applied to direct sums of symmetric operators and Sturm-Liouville operators with operator potentials.

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