The infinite-dimensional Sylvester differential equation and periodic output regulation

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In this presentation we study the infinite-dimensional Sylvester differential equation

$$\dot{\Sigma}(t) + \Sigma(t)B(t) = A(t)\Sigma(t) + C(t), \qquad \Sigma(t_0) = \Sigma_0$$

where $(A(t), \mathcal{D}(A))$ and $(B(t), \mathcal{D}(B))$ are families of unbounded operators on Banach spaces X and Y, respectively, and $C(\cdot) \in C(\mathbb{R}, \mathcal{L}_s(Y, X))$. Our main interests are the results on the solvability of the equation and in particular the conditions for the existence of a unique periodic solution when the families $(A(t), \mathcal{D}(A))$ and $(B(t), \mathcal{D}(B))$ and the operator-valued function $C(\cdot)$ are periodic.

These results have an application in the output regulation of a distributed parameter system with a nonautonomous signal generator

$$\dot{v}(t) = S(t)v(t), \qquad v(0) = v_0,$$

where $S(\cdot) \in C^1(\mathbb{R}, \mathbb{C}^{q\times q})$ is a periodic function. We show that the controllers solving the periodic output regulation problem can be characterized using the properties of the periodic solution of a Sylvester differential equation. This generalizes the results related to the control of distributed parameter systems with autonomous exosystems where the controllers solving the output regulation problem can be characterized by the solvability of certain constrained Sylvester equations.

The talk is based on joint work with S. Pohjolainen.