Hypercyclic dynamics of translation operators and universal harmonic functions of slow growth

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A harmonic function H on $\mathbb{R}^N$ is said to be universal with respect to translations if the set of translates $\{H(\cdot + a)\ ;\ a \in \mathbb{R}^N\}$ is dense in the space of all harmonic functions on $\mathbb{R}^N$ with the topology of local uniform convergence, that is, the topology of uniform convergence on compact subsets of $\mathbb{R}^N$, also called compact-open topology. Dzagnidze (1964) showed that there are universal harmonic functions on $\mathbb{R}^N$. Recently, Armitage (2005) proved that universal harmonic functions can also have slow growth. More precisely, given any $\phi : [0, +\infty[\to]0, +\infty[$, a continuous increasing function such that

$$\lim_{t\to\infty} \frac{\log \phi(t)}{(\log t)^2} = +\infty, \qquad (1)$$

then there is a universal harmonic function H on $\mathbb{R}^N$ that satisfies $|H(x)| \leq \phi(||x||)$ for all $x \in \mathbb{R}^N$. Armitage asked whether the condition (1) can be relaxed: Is it true that, for every $N > 2$, one can find universal harmonic functions on $\mathbb{R}^N$ with arbitrarily slow transcendental growth? In other words, can the exponent 2 of $\log t$ be reduced to 1? We answer positively this question.

Our techniques depend on the analysis of the hypercyclic behaviour of the translation operator $T_a : X \to X$, defined on certain Banach spaces X consisting of harmonic functions of slow growth.

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