Locally definite normal operators in Krein spaces

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Let N be a bounded normal operator in the Krein space $(\mathcal{H}, [\cdot,\cdot])$, i.e. $NN^+ = N^+N$, where N^+ denotes the Krein space adjoint of N. We say that a number $\lambda \in \sigma_{ap}(N)$ is a *spectral point of positive type* of N if for every sequence $(x_n) \subset \mathcal{H}$ with $\|x_n\| = 1$ for all $n \in \mathbb{N}$ and $(N-\lambda)x_n \to 0$ as $n \to \infty$ we have

$$\liminf_{n\to\infty}\, [x_n, x_n] > 0.$$

In the paper [1] the authors showed that there exists a local spectral function for the *selfadjoint* operator N on an interval Δ if every spectral point of N in Δ is of positive type. For normal operators we prove the following theorem.

Theorem. *Assume that $\sigma(\operatorname{Re} N) \subset \mathbb{R}$, $\sigma(\operatorname{Im} N) \subset \mathbb{R}$ and that there exist $M > 0$, $n \in \mathbb{N}$ and an open neighborhood $\mathcal{U}$ of $\sigma(\operatorname{Im} N)$ in $\mathbb{C}$ such that*

$$\|(\operatorname{Im} N - \lambda)^{-1}\| \;\le\; M\,|\operatorname{Im}\lambda|^{-n} \qquad \text{for all } \lambda \in \mathcal{U}\setminus\mathbb{R}.$$

If $R \subset \mathbb{C}$ is a closed rectangle such that every spectral point of N in R is of positive type with respect to N, then N has a local spectral function on R.

This talk is based on a joint work with C. Trunk and V. Strauss.

References

[1] H. Langer, A. Markus, V. Matsaev: Locally definite operators in indefinite inner product spaces. Math. Ann. **308** (1997), 405–424.