Dixmier traces of operators on Banach and Hilbert spaces

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Traces of operators on a Hilbert space were first considered by J. von Neumann in 1932. In collaboration with F. J. Murray, he extended this concept to $W^\star$-algebras. Another direction of development led to a theory of traces of operators on Banach spaces (R. Schatten, A. Grothendieck, and A. Pietsch).

Originally, the definition of a trace was designed as a generalization of the classical trace of a square matrix. In a next step, traces were considered as unitarily invariant positive linear functionals. Taking this point of view, Dixmier (1966) constructed exotic traces by using a modification of Banach limits. Surprisingly, these strange objects became a powerful tool in Connes's *"Noncommutative Geometry."*

The lecture gives a new approach to the theory of Dixmier traces that is based on results obtained about 20 years ago. I would like to show that Banach space techniques are useful also in the setting of Hilbert spaces.

References

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