# One-dimensional and multidimensional spectral order

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As shown by Kadison, the set $\mathcal{S}$ of all bounded selfadjoint operators on a complex Hilbert space $\mathcal{H}$ is an anti-lattice, which means that for $A, B \in \mathcal{S}$, a greatest lower bound for $A$ and $B$ exists with respect to the usual ordering "$\leqslant$" in $\mathcal{S}$ if and only if $A$ and $B$ are comparable (cf. [1]). A little bit earlier, Sherman proved that if the set of all selfadjoint elements of a $C^*$-algebra $\mathcal{A}$ of bounded linear operators on $\mathcal{H}$ is lattice ordered by "$\leqslant$", then $\mathcal{A}$ is commutative (cf. [3]). To overcome these disadvantages of the partial order "$\leqslant$", Olson introduced in 1971 the so called *spectral order*, which is denoted by "$\preccurlyeq$". He proved, among other things, that the set of all selfadjoint elements of a von Neumann algebra of bounded linear operators on $\mathcal{H}$ is a conditionally complete lattice with respect to the spectral order (cf. [2]).

We extend the spectral order to the case of $n$-tuples of spectrally commuting selfadjoint operators and investigate its properties.

## References

[1] R. V. Kadison, Order properties of bounded self-adjoint operators, *Proc. Amer. Math. Soc.* **2** (1951), 505-510.

[2] M. P. Olson, The selfadjoint operators of a von Neumann algebra form a conditionally complete lattice, *Proc. Amer. Math. Soc.* **28** (1971), 537-544.

[3] S. Sherman, Order in operator algebras, *Amer. J. Math.* **73** (1951), 227-232.

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