Convergence of the Dirichlet-to-Neumann map on thin branched manifolds

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We consider a family of manifolds $(X_\varepsilon)_\varepsilon$ that shrinks to a metric graph X_0 as $\varepsilon \to 0$, i.e., a topological graph where each edge is assigned a length. A simple example is given by the (smoothed) surface of the ε-tubular neighbourhood of X_0. Let Y_0 be the set of vertices of degree 1, and Y_ε the corresponding boundary of X_ε. Using boundary triples based on first order Sobolev spaces, we can define the Dirichlet-to-Neumann map $\Lambda_\varepsilon(z)$ associated to the boundary Y_ε in X_ε and a Laplace-type operator $\Delta_\varepsilon \geq 0$ for $\varepsilon \geq 0$. In particular, for suitable φ on Y_ε, we let $\Lambda_\varepsilon(z)\varphi$ be the "normal" derivative of the Dirichlet solution h of $(\Delta_\varepsilon - z)h = 0$ with boundary data φ.

Our main result is the convergence of the Dirichlet-to-Neumann map $\Lambda_\varepsilon(z)$ to $\Lambda_0(z)$ in a suitable sense, since the operators act in different spaces. In this way, we can approximately calculate the Dirichlet-to-Neumann operator on the more complicated space X_ε in terms of the simpler one on X_0. The talk is based on a joint work with J. Behrndt.