Higher rank numerical ranges

L. Rodman

The rank k numerical range of a linear bounded operator A on a complex Hilbert space $\mathcal{H}$ is defined as follows:

$$\begin{aligned}\Lambda_k(A) = \{\lambda \in \mathbf{C} \,:\, PAP &= \lambda P \text{ for some rank } k \\ &\quad \text{orthogonal projection } P \text{ on } \mathcal{H}\}.\end{aligned}$$

Here k is a positive integer smaller than the dimension of $\mathcal{H}$. If $k = 1$, then the standard numerical range of A is obtained. Besides mathematical interest, the study of higher rank numerical ranges is motivated by applications in quantum error correction.

Basic properties of $\Lambda_k(A)$ will be discussed, such as convexity. In particular, under appropriate hypotheses, a description will be given of linear preservers of rank k numerical ranges, i.e., linear maps ϕ such that $\Lambda_k(A) = \Lambda_k(\phi(A))$ for all operators A (this result was obtained jointly with S. Clark, C.-K. Li, J. Mahle).