On well-posedness and meromorphic solutions of the Korteweg-de Vries equation with non-decaying initial data supported on a left half-line

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The talk is concerned with the initial value problem (Cauchy problem) for the Korteweg-de Vries (KdV) equation on the domain $-\infty < x < \infty, t \geq 0$:

$$\begin{cases} \partial_t V - 6V\partial_x V + \partial_x^3 V = 0 \\ V(x,0) = V_0(x) \end{cases} \tag{1}$$

with real initial data V_0 vanishing on $(0,\infty)$ and essentially arbitrary on $(-\infty,0)$. We show that if the spectrum of the half line Schrodinger operator

$$H = -\partial_x^2 + V_0(x), \quad u(-0) = 0 \tag{2}$$

is bounded below and has a non-trivial absolutely continuous component then the problem (1) is well-posed. Moreover, under the KdV flow any such initial profile $V_0(x)$ (no matter how rough and without any decay assumption) instantaneously evolves into a meromorphic function in x on the whole complex plane with no real poles. Our treatment is based on a suitable modification of the inverse scattering transform and a detailed investigation of the Titchmarsh-Weyl $m-$function associated with (2). As by-product, we improve some related results of others.