Linear operators with two invariant subspaces: Swiss Cheese Theorem and an application to control theory

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Through recent developments in the representation theory of finite dimensional algebras, refined invariants, new classification results, and detailed descriptions have been obtained for systems consisting of vector spaces, linear operators, and invariant subspaces, see in particular [RS, Section (0.1)].

In this talk we focus on the case of pairs of invariant subspaces, more precisely, we consider systems (V, T, U_1, U_2) where V is a finite dimensional vector space, T a linear operator acting on V nilpotently, and where $U_1 \subset U_2 \subset V$ are two T-invariant subspaces.

The complexity of the problem of classifying such systems depends on the upper bound n on the nilpotency index: For $n \leq 3$, there are only finitely many indecomposable systems, while the problem is considered infeasible for $n \geq 5$. The case where $n = 4$ is controlled by a finite dimensional algebra of tubular type $\mathbb{E}_7$. We determine the set of dimension triples

$$\{(\dim U_1, \dim U_2/U_1, \dim V/U_2) : (V, T, U_1, U_2) \text{ indecomposable}\} \subset \mathbb{R}^3,$$

which is contained in a cylinder with axis $(1, 1, 1)\mathbb{R}$, and show that the corresponding dual complex is unbounded, connected and simply connected but surprisingly contains holes.

In control theory, invariant subspaces arise as the controllable and the unobservable states in a linear time-invariant control system. Their quotient is the minimal realization in the Kalman decomposition, which we discuss for typical systems arising in this talk.

This is a report on joint work with A. Moore.

Reference

[RS] C. M. Ringel and M. Schmidmeier, *Invariant Subspaces of Nilpotent Linear Operators. I,* Journal für die reine und angewandte Mathematik **614** (2008), 1-52.
`http://www.atypon-link.com/WDG/doi/abs/10.1515/CRELLE.2008.001`