The Riemann - Hilbert boundary value problem with a countable set of coefficient and two - side curling at infinity of order less then 1/2

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Let L be the real axis at the plane of the complex variable z, $D = \{z, \mathrm{Im} z > 0\}$. We need to find an analytic function $F(z)$ in the domain D by the boundary condition $a(t)\mathrm{Re}F(t) - b(t)\mathrm{Im}F(t) = c(t), \quad t \in L$, where $a(t)$, $b(t)$, $c(t)$ are given real functions on L, continuous everywhere except of ordinary discontinuity points t_k, $k = \pm 1, \pm 2, \cdots$, $\lim\limits_{k\to\pm\infty} t_k = \pm\infty$.

Let us denote $G(t) = a(t) - ib(t)$ and let $\nu(t) = \arg G(t)$ be the branch defined on every interval of continuity of coefficients, so that the numbers $\delta_k = \nu(t_k+0) - \nu(t_k-0)$ satisfy conditions $0 \le \delta_k < 2\pi$, $k = \pm 1, \pm 2, \cdots$. Now let us denote $\kappa_k = \delta_k/\pi - [\delta_k/\pi]$, $k = \pm 1, \pm 2, \cdots$, where $0 \le \kappa_k < 1$. Let's consider that $\varphi_1(t)$ - the continuous component of function $\nu(t)$, satisfies to a condition $\varphi_1(t) = \nu^- t^\rho + \tilde{\nu}(t), t > 0, \quad \varphi_1(t) = \nu^+ |t|^\rho + \tilde{\nu}(t), t < 0$, where ν^-, ν^+, ρ are constant, $0 < \rho < 1/2$, $\tilde{\nu}(t) \in H_L(\mu)$, $0 < \mu \le 1$. We denote $n^*_-(x) = \sum\limits_{j=1}^{k-1} \kappa_{-j}, -t_{-k+1} \le x < -t_{-k}$, $n^*_+(x) = \sum\limits_{j=1}^{k-1} \kappa_j, t_{k-1} \le x < t_k$. Assume that the numbers t_j, κ_j are satisfy the conditions $n^*_+(x) = \Delta_+ x^{\kappa_+} + o(x^{\kappa_+}), \quad n^*_-(x) = \Delta_- x^{\kappa_-} + o(x^{\kappa_-}), \quad x \to +\infty$, where κ_+, κ_-, Δ_+, Δ_- are positive constants $\kappa_- < 1/2$, $\kappa_+ < 1/2$. We set that $n^*_-(-t_{-k}) - \Delta_-(-t_{-k})^{\kappa_-} = p_{-k}, \quad n^*_+(t_k) - \Delta_+(t_k)^{\kappa_+} = p_k$ where we choose the numbers p_{-k}, p_k, t_{-k}, t_k so that the inequalities $p_{-k} = -n^*_-(-t_{-k}) + \Delta_-(-t_{-(k+1)})^{\kappa_-}$, $p_k = -n^*_+(t_k) + \Delta_+(t_{k+1})^{\kappa_+}$ are fulfilled. At the given restrictions full research of resolvability of a homogeneous problem in one class is conducted. The formula of the common decision of an unhomogeneous problem is proved.

The talk is based on a joint work with R. Salimov.