On invariant subspaces of an operator, some exterior power of which is positive

P. Sharma

Let a linear operator A act in the finite dimensional space $\mathbb{R}^n$. In this case we can define its jth ($j = 2, \ \ldots, \ n$) exterior power $\wedge^j A$, which acts in the finite-dimensional space $\mathbb{R}^{C_n^j}$. Let for some j_0 ($1 \leq j_0 \leq n$) the operator $\wedge^{j_0} A$ leave invariant a pointed, closed and full convex cone $K_{j_0} \subset \mathbb{R}^{C_n^{j_0}}$. Let, in addition, the only nonempty subset of $\partial(K_{j_0})$, which is left invariant by $\wedge^{j_0} A$, be $\{0\}$. Then under some additional conditions we can prove, that the initial operator A leaves invariant a cone $\mathcal{T}(K_{j_0})$ of rank j_0, i.e. a closed subset $\mathcal{T}(K_{j_0}) \subset \mathbb{R}^n$, which satisfy the following conditions:

1) For every $x \in \mathcal{T}(K_{j_0})$, $\alpha \in \mathbb{R}$ the element $\alpha x \in \mathcal{T}(K_{j_0})$.

2) There is at least one j_0-dimensional subspace and no higher dimensional subspaces in $\mathcal{T}(K_{j_0})$.

Moreover, the operator A has a unique invariant j_0-dimensional subspace M, lying in $A\mathcal{T}(K_{j_0})$. The restriction of the operator A to the subspace M is invertible. If another finite-dimensional subspace $L \subset A\mathcal{T}(K_{j_0})$ is also invariant for the operator A, then $L \subset M$.

Some estimates of the spectral gap of the operator A are also obtained.

The talk is based on a joint work with O. Kushel.

References.

1. M.A. Krasnosel'skii, Je.A. Lifshits, A.V. Sobolev, *Positive Linear Systems: The method of positive operators.* Berlin: Helderman Verlag, Sigma Series in Applied Mathematics, 1989.