On the spectrum of some class of Jacobi operators in Krein space

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The class of three-diagonal Jacobi matrix with exponentially increasing elements is considered. Under some assumptions the matrix corresponds to unbounded self-adjoint operator L in the weighted space $l_2(\omega)$ with the scalar product $(x, y) = \sum_{k=1}^{\infty} \omega_k x_k \overline{y_k}$. The weight can arise indefinite metric in some cases.

We proved that the eigenvalue problem for this operator is equivalent to the eigenvalue problem of Sturm–Liouville operator with discrete self-similar weight. The asymptotic formulas for eigenvalues are obtained. These formulas differ for cases of definite and indefinite metrics.

Theorem 1) Spectrum of operator L is discrete. If L is self-adjoint in Hilbert space then all eigenvalues are positive and simple. There exists a positive number c such that eigenvalues λ_n enumerated in increasing order satisfy the following asymptotic formula

$$\lambda_n = cq^n(1 + o(1)) \quad (n \to +\infty).$$

2) If L is self-adjoint in Krein space then all eigenvalues are simple. There exists a positive number c such that positive eigenvalues λ_n enumerated in increasing order satisfy the asymptotic formula

$$\lambda_{n+1} = cq^{2n}(1 + o(1)) \quad (n \to +\infty)$$

and negative eigenvalues λ_{-n} enumerated in increasing order by absolute values satisfy the following asymptotic formula

$$\lambda_{-(n+2)} = -cq^{2n+1}(1 + o(1)) \quad (n \to +\infty).$$

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