On perturbations of self-adjoint or normal operators

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In the first part of the talk we shall deal with perturbations of a self-adjoint or normal operator T with discrete spectrum. There are results which allow to compare the eigenvalue counting functions $n(r, T)$ and $n(r, T + B)$, provided that the perturbation B is relatively compact or p-subordinated to T, i.e.

$$\|Bx\| \leqslant \text{const}\|Tx\|^p \,\|x\|^{1-p}, \quad \text{for } x \in \mathcal{D}(T) \tag{1}$$

with some $p < 1$. There are results (obtained by M.Keldysh, F.Brauder, S.Agmon, V.Lidskii, I.Gohberg and M.Krein, A.Markus and V.Matsaev, V.Kaznelson, M.Agranovich and others) which allow to assert that the eigen- and associated vectors of the perturbed operator $T + B$ form a basis for Abel summubility method or an uncoditional basis, provided that some relations between p and the order of growth of $n(r, T)$ hold. We shall present similar results replacing the condition (1) by a weaker assumption

$$\|B\varphi_k\| \leqslant \text{const}|\mu_k|^p,$$

where $\{\varphi_k\}_{k=1}^{\infty}$ is an orthonormal system of the eigenvectors of T corresponding to the eigenvalues $\{\mu_k\}_{k=1}^{\infty}$. The advantage of the last condition will be demonstrated by concrete examples.

In the second part we will discuss perturbations of a self-adjoint operator T whose spectrum consists of infinitely many components $\{\sigma_k\}_{k=1}^{\infty}$ separated by gaps: $\text{dist}(\sigma_k, \sigma_{k+1}) \geqslant \text{const}$.